Reply to the comment in quant-ph/0609028 on controlled teleportation

Chui-Ping Yang

Department of Physics, Carnegie Mellon university, Pittsburgh, PA 15213

Abstract

This is to reply to the comment of Kenigsberg and Mor on our previous work "Efficient many-party controlled teleportation of multi-qubit quantum information via entanglement"[Phys. Rev. A 70, 022329 (2004)].

Kenigsberg and Mor discussed controlled teleportation in their recent paper [1]. They made some comments on our previous work about "Efficient many-party controlled teleportation of multi-qubit quantum information via entanglement" [2]. However, we note that their comments are not correct. The reasons are as follows.

First, the state of Eq. (3) in Ref. [1]

$$\otimes_{i=1}^{m} \left|\phi^{+}\right\rangle_{AB(i)} \otimes \left|\phi^{+}\right\rangle_{AC} + \otimes_{i=1}^{m} \left|\phi^{-}\right\rangle_{AB(i)} \otimes \left|\psi^{+}\right\rangle_{AC}$$

is different from the state of Eq. (2) in Ref. [2].

Second, they claimed that Alice and Bob can distill the following mixed state described by Eq. (4) in Ref. [1]

$$\otimes_{i=1}^{m} \left|\phi^{+}\right\rangle \left\langle\phi^{+}\right|_{AB(i)} \otimes 1_A + \otimes_{i=1}^{m} \left|\phi^{-}\right\rangle \left\langle\phi^{-}\right|_{AB(i)} \otimes 1_A$$

and thus Alice can teleport $(m-1)$-qubit state to Bob. However, as a matter of fact, even based on the state of Eq. (3) used in Ref. [1], one cannot obtain the above mixed state, after tracing over Carlo's qubit. To see this, let us rewrite the above state of Eq. (3) in Ref. [1] as follows:

$$\begin{aligned}
&\otimes_{i=1}^{m} \left|\phi^{+}\right\rangle_{AB(i)} \otimes \left|\phi^{+}\right\rangle_{AC} + \otimes_{i=1}^{m} \left|\phi^{-}\right\rangle_{AB(i)} \otimes \left|\psi^{+}\right\rangle_{AC} \\
&= \frac{1}{\sqrt{2}} \left[\otimes_{i=1}^{m} \left|\phi^{+}\right\rangle_{AB(i)} \otimes \left(|00\rangle_{AC} + |11\rangle_{AC}\right) + \otimes_{i=1}^{m} \left|\phi^{-}\right\rangle_{AB(i)} \otimes \left(|01\rangle_{AC} + |10\rangle_{AC}\right)\right] \\
&= \frac{1}{\sqrt{2}} \left[\left(\otimes_{i=1}^{m} \left|\phi^{+}\right\rangle_{AB(i)} |0\rangle_A + \otimes_{i=1}^{m} \left|\phi^{-}\right\rangle_{AB(i)} |1\rangle_A\right) \otimes |0\rangle_C \right. \\
&\quad \left. + \left(\otimes_{i=1}^{m} \left|\phi^{+}\right\rangle_{AB(i)} |1\rangle_A + \otimes_{i=1}^{m} \left|\phi^{-}\right\rangle_{AB(i)} |0\rangle_A\right) \otimes |1\rangle_C\right]
\end{aligned}$$

One can check that after tracing over Carlo's qubit from the state of Eq. (3) in Ref. [1], the density operator for the m EPR pairs shared by Alice and Bob and the additional qubit held by Alice is

$$\begin{aligned}
&\left(\otimes_{i=1}^{m} \left|\phi^{+}\right\rangle \left\langle\phi^{+}\right|_{AB(i)} + \otimes_{i=1}^{m} \left|\phi^{-}\right\rangle \left\langle\phi^{-}\right|_{AB(i)}\right) \left(|0\rangle_A \langle 0| + |1\rangle_A \langle 1|\right) \\
&+ \left(\otimes_{i=1}^{m} \left|\phi^{+}\right\rangle \left\langle\phi^{-}\right|_{AB(i)} + \otimes_{i=1}^{m} \left|\phi^{-}\right\rangle \left\langle\phi^{+}\right|_{AB(i)}\right) \left(|0\rangle_A \langle 1| + |1\rangle_A \langle 0|\right),
\end{aligned}$$

which is obviously different from the above mixed state described by Eq. (4) in Ref. [1].

Last, we think that they might doubt the entangled state of Eq. (2) in Ref. [2], which they wrote as the state of Eq. (3) in Ref. [1] by a mistake. However, it is easy to check that the mixed state of Eq. (4) in Ref. [1] still cannot be created from the state of Eq. (2) in Ref. [2], by tracing over Carlo's qubit (even for the case when a Hadamard gate is performed by Alice on her GHZ or EPR qubit).

References

[1] D. Kenigsberg and T. Mor, quant-ph/0609028

[2] C. P. Yang, S. I. Chu, and S. Han, Phys. Rev. A 70, 022329 (2004)